SAT Backtracking with Interactive User

SMITE Seminar, Belo Horizonte, Brazil

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Acknowledgements

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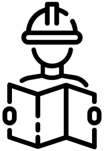
Introduction



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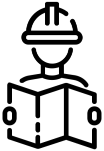

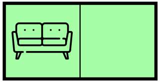

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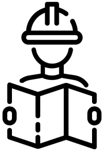

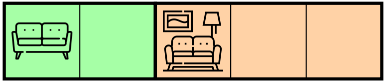

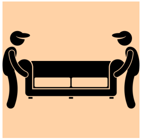

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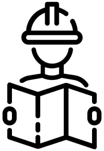

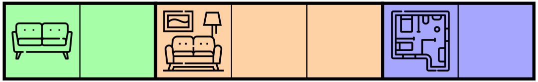

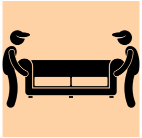

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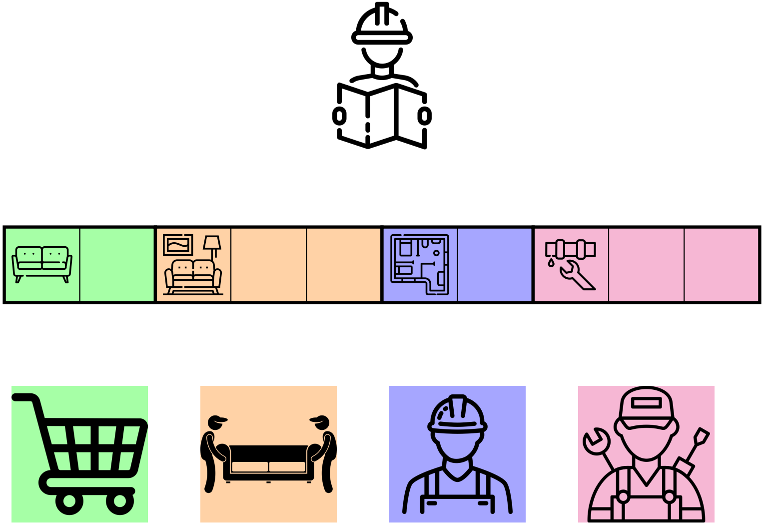
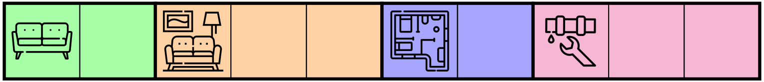
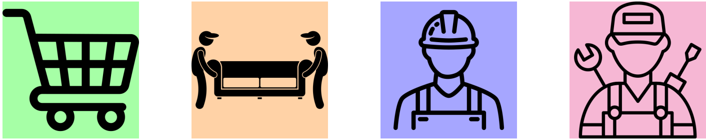


Part I: Chronological Backtracking

The Movers Have a Suggestion





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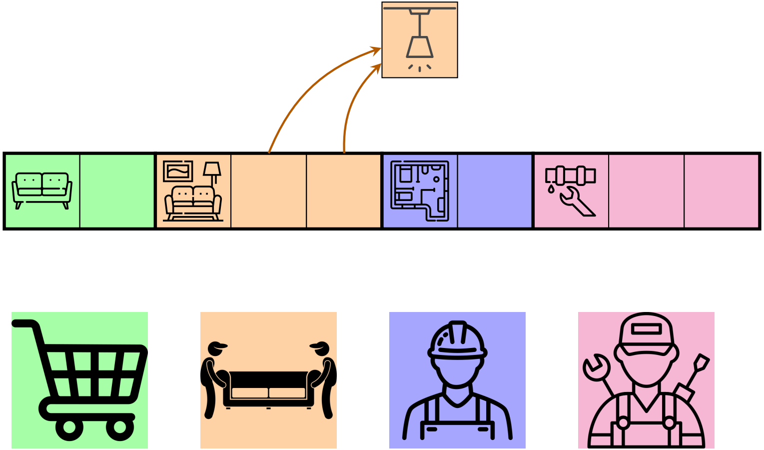

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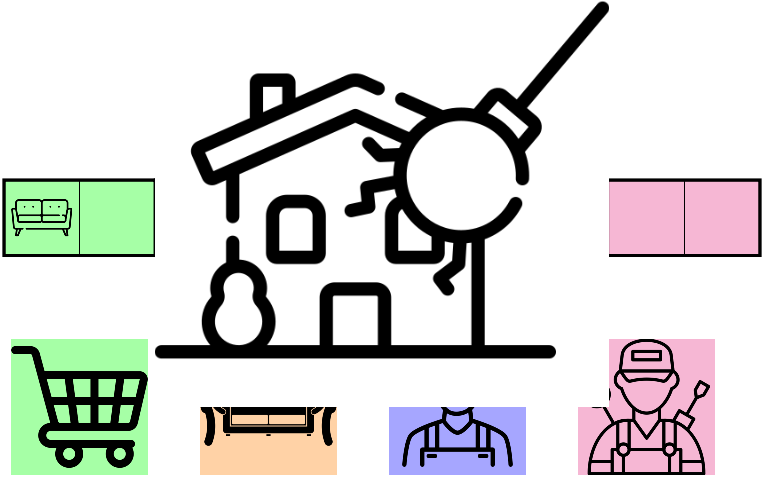

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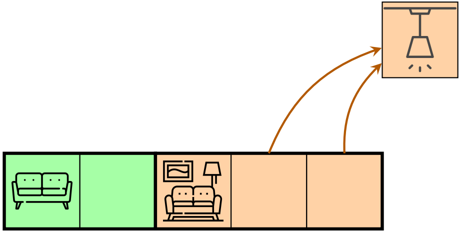

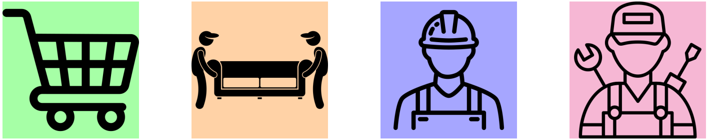

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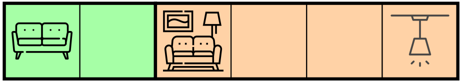

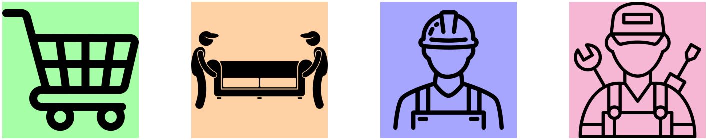

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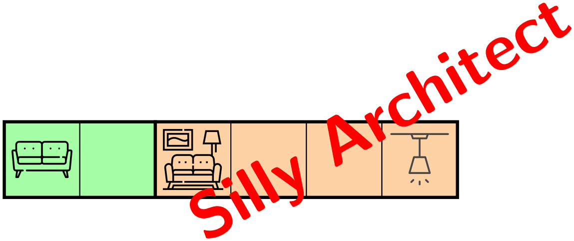
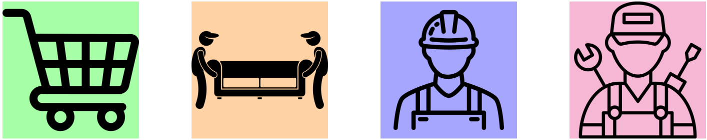
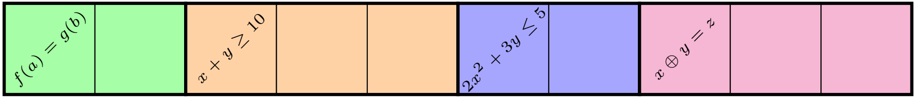




SMT Architecture

SAT Solver



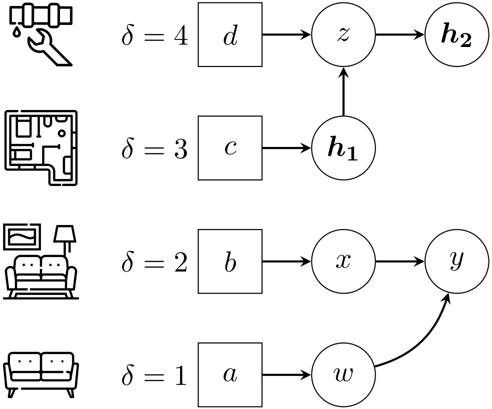
UF
(Uninterpreted Functions)

LRA
(Linear Real Arithmetic)

NRA
(Non-linear Real Arithmetic)

BV
(Bit Vectors)

Non-Chronological Backtracking: An Incremental Literal



$$
\begin{aligned}
C_1 &= \underline{w} \lor \underline{\neg a} \\
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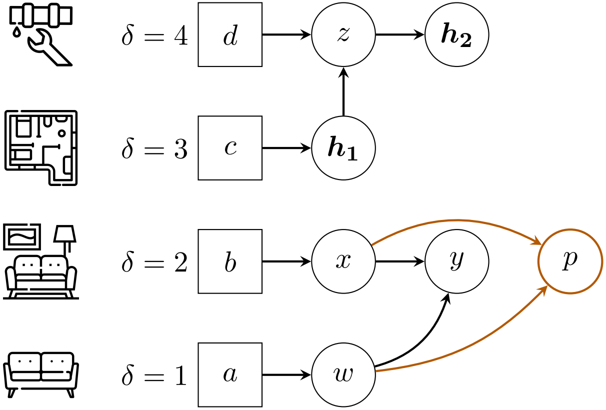

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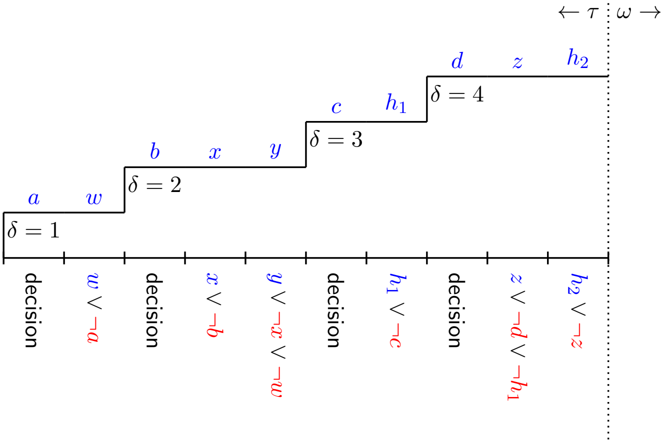
Why it Breaks [NR18, MB19]



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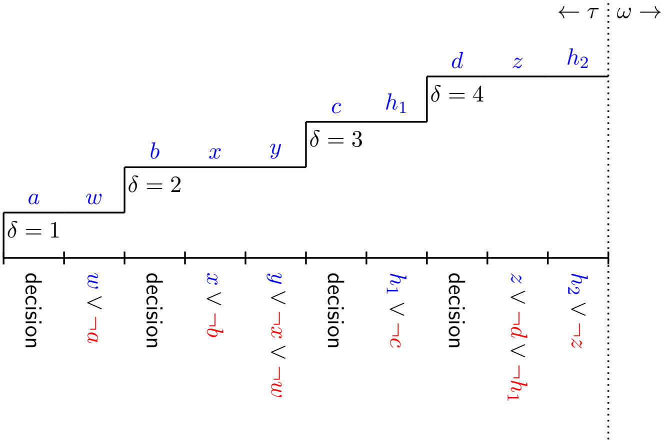

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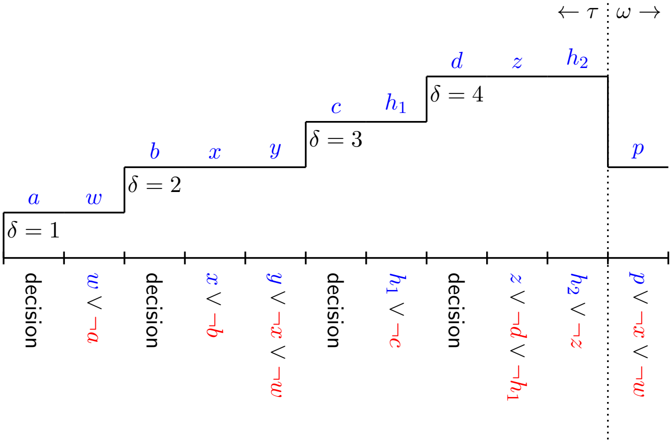

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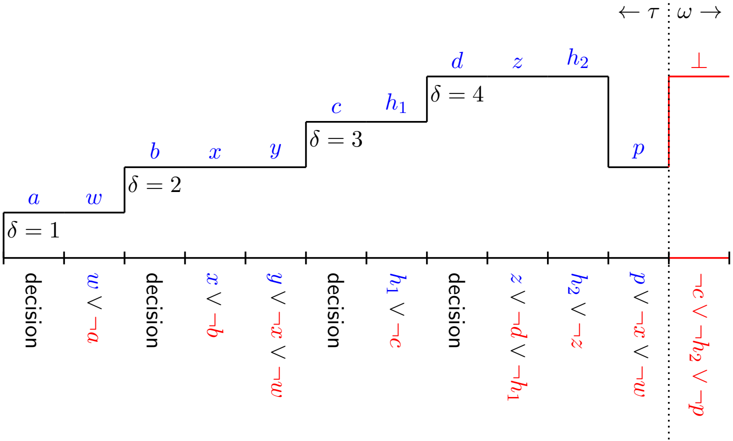

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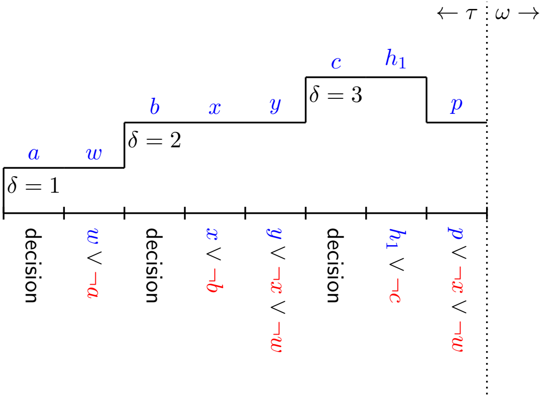

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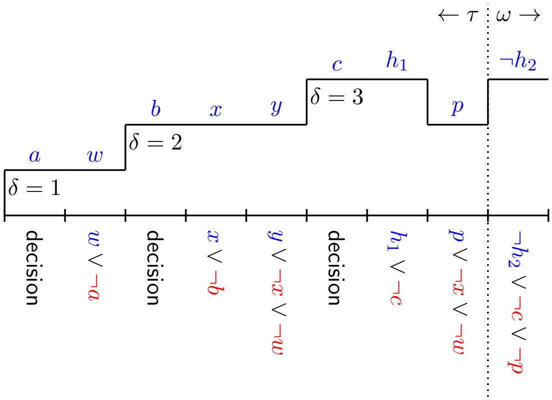

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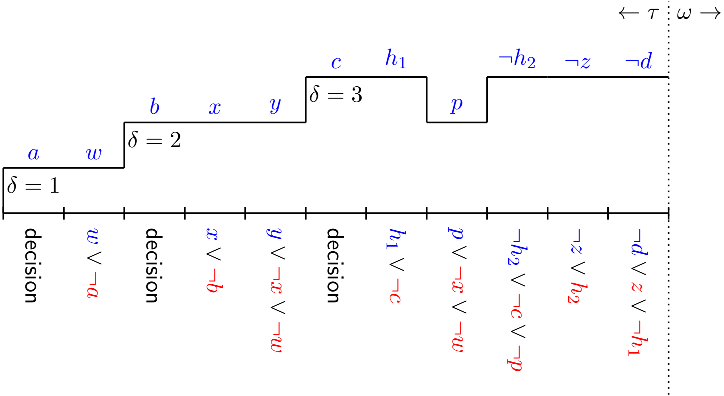

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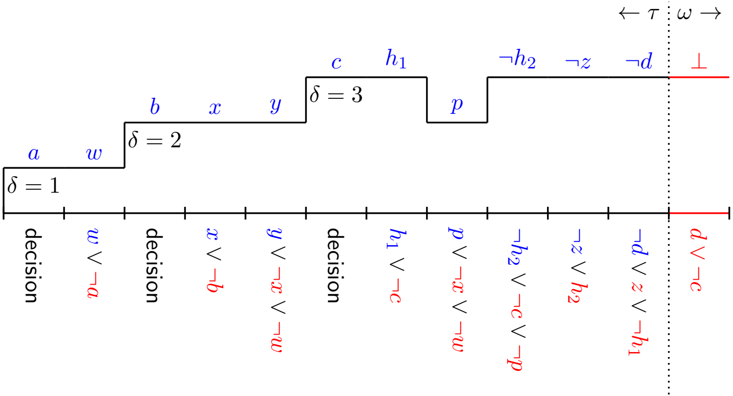

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C_7 &= a \lor \neg c \lor \neg d \\
C_8 &= p \lor \neg x \lor \neg w \\
C_9 &= \neg h_2 \lor \neg p \lor \neg c \\
C_{10} &= d \lor \neg c
\end{aligned}
$$

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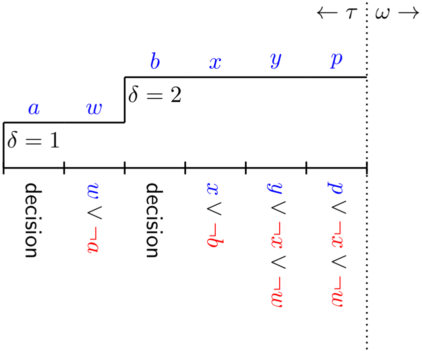

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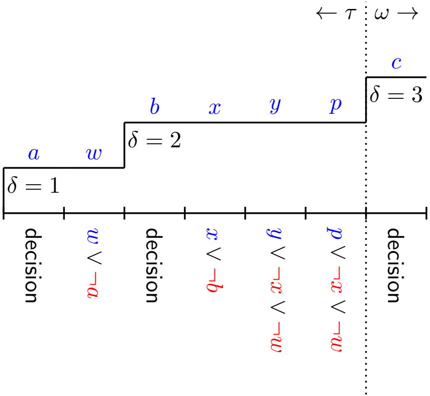

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\end{aligned}
$$

We will miss the implication of $\neg h_2$ by C_9 because the watches are wrong.

Watched Literal Invariants [CFK24]

Let $\pi = \tau \cdot \omega$ be the current partial assignment, where τ is the propagated set.

Invariant (Watched Literals)

For each clause C in the formula watched by $c_1, c_2 \in C$, it must hold that:

$$\neg c_1 \in \tau \Rightarrow c_2 \in \pi$$

or equivalently:

$$\textit{propagated}(\neg c_1) \Rightarrow \textit{satisfied}(c_2)$$

Watched Literal Invariants [CFK24]

Let $\pi = \tau \cdot \omega$ be the current partial assignment, where τ is the propagated set.
Let $\delta(\ell)$ be the decision level of literal ℓ in π.
Implicitly, in NCB, we have:

Invariant (Backtrack Compatible Watched Literals)

For each clause C in the formula watched by $c_1, c_2 \in C$, it must hold that:

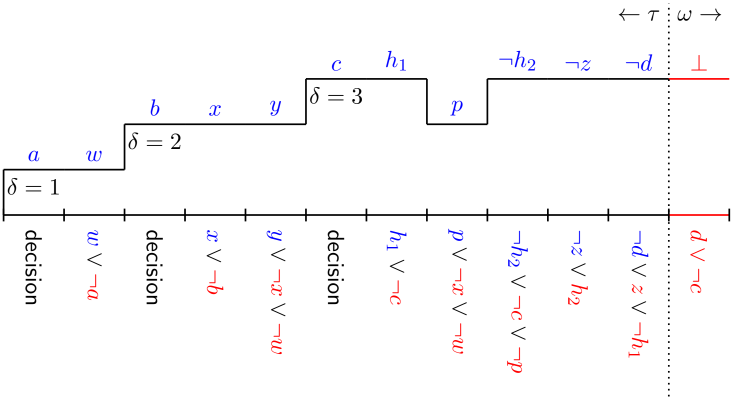
$$\neg c_1 \in \tau \Rightarrow \big(c_2 \in \pi \wedge \delta(c_1) \geq \delta(c_2)\big)$$

or equivalently:

$$\textit{propagated}(\neg c_1) \Rightarrow \big(\textit{satisfied}(c_2) \wedge c_2 \textit{ is not higher than } c_1\big)$$

Solution: Trail Fixing [NR18, MB19]

When backtracking to a level δ, we need to repropagate all out-of-order literals.



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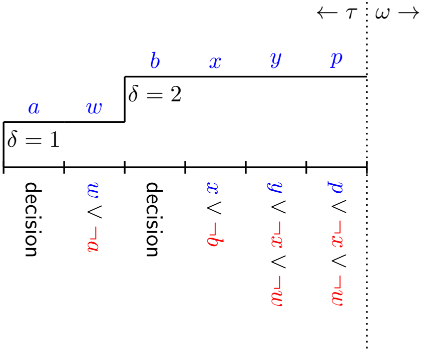

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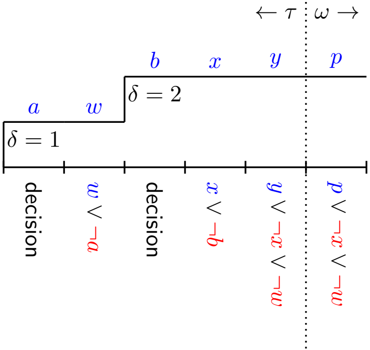

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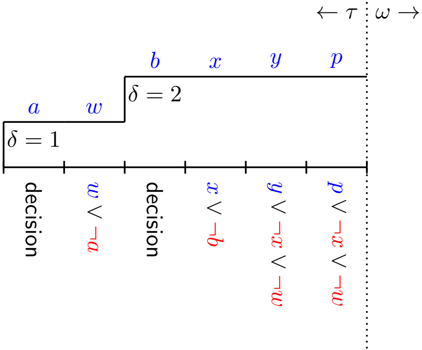

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C_4 &= \underline{\neg c} \vee \underline{h_1} \\
C_5 &= \underline{\neg d} \vee \underline{z} \vee \neg h_1 \\
C_6 &= \underline{\neg z} \vee \underline{h_2} \\
C_7 &= \underline{a} \vee \underline{\neg c} \vee \neg d \\
C_8 &= \underline{p} \vee \underline{\neg x} \vee \neg w \\
C_9 &= \underline{\neg h_2} \vee \underline{\neg c} \vee \neg p \\
C_{10} &= \underline{\neg c} \vee \underline{d}
\end{aligned}
$$

Watched Literal Invariants (Restorable)

Let $\pi = \tau \cdot \omega$ be the current partial assignment, where τ is the propagated set.
Let $pos(\ell)$ be position of a literal ℓ in π and π^d be the ordered set of decisions.
We can can strengthen the invariant to:

Invariant (Restorable Watched Literals)

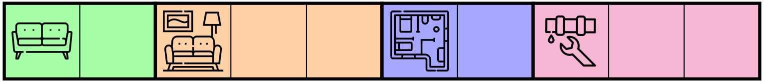
For each clause C in the formula watched by $c_1, c_2 \in C$, it must hold that:

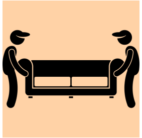
$$\neg c_1 \in \tau \Rightarrow \left(c_2 \in \pi \wedge pos(c_1) \geq pos(\pi^d[\delta(c_2)])\right)$$

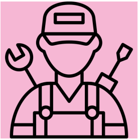
or equivalently:

$$\textit{propagated}(\neg c_1) \Rightarrow \left(\textit{satisfied}(c_2) \wedge c_1 \textit{ is after the decision of } c_2\right)$$

Cool! No Need to Demolish the House





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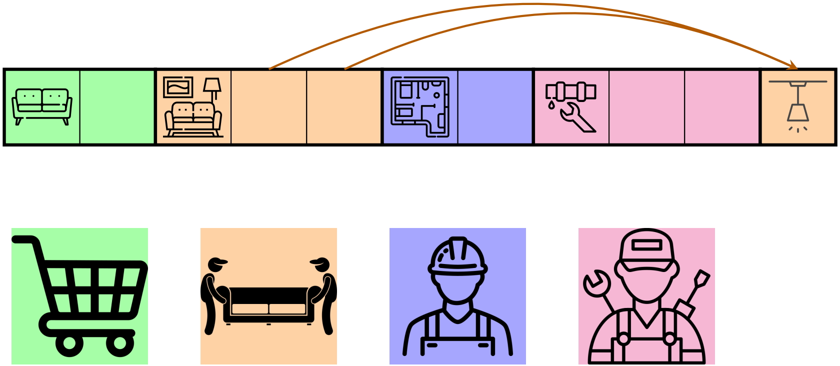
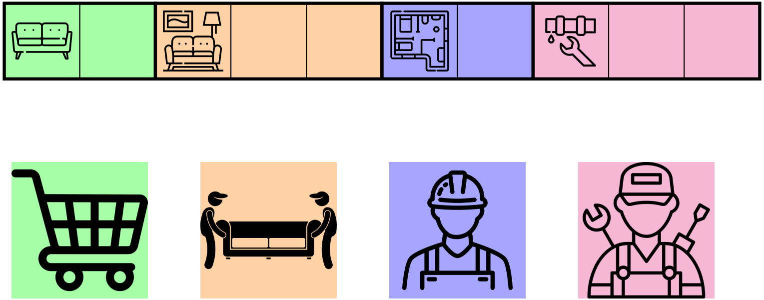


Part II: Lazy Chronological Backtracking

The Electrician Thought of it First



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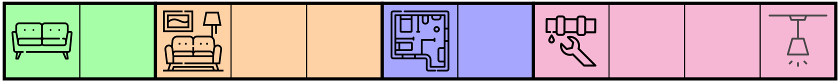

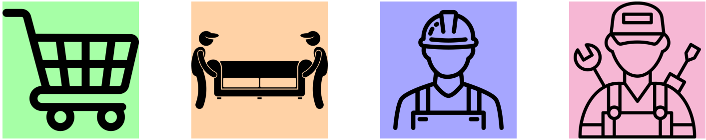

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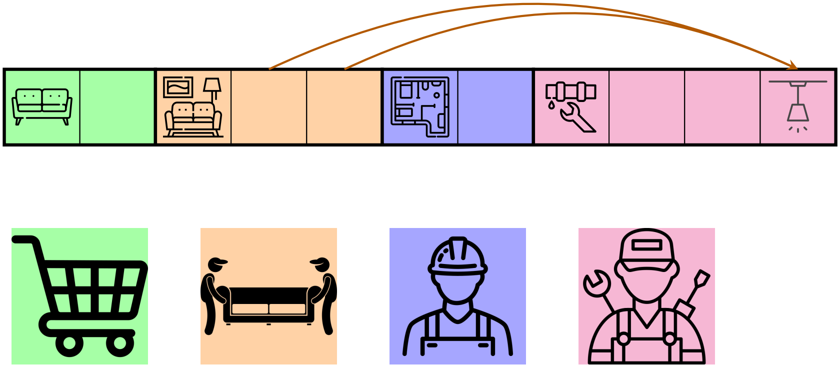

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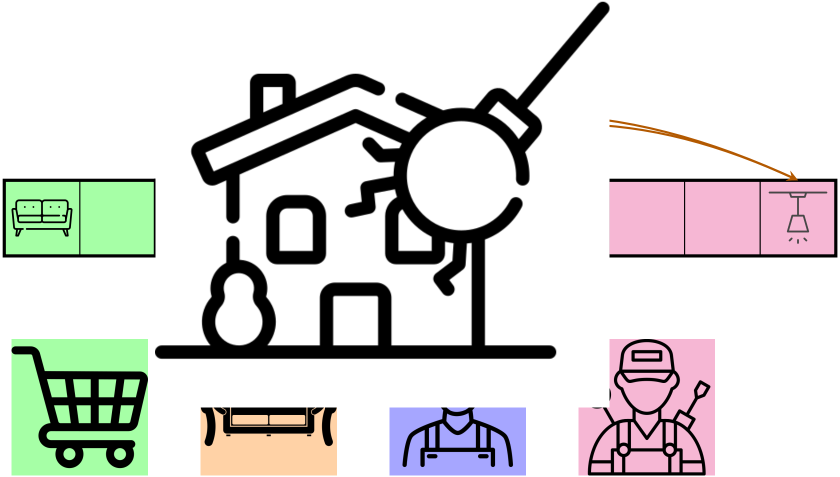

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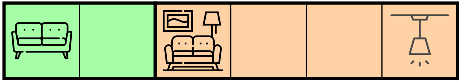
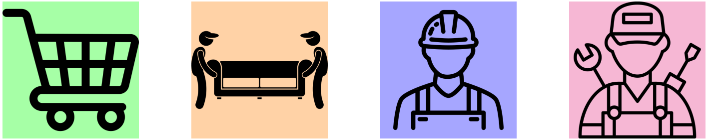
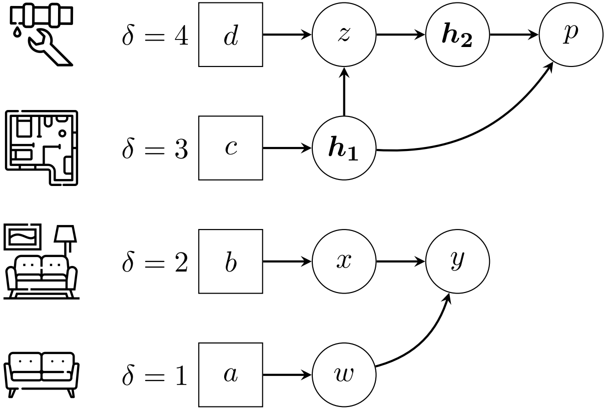




Missed Lower Implication



$$
\begin{aligned}
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C_2 &= \underline{x} \lor \underline{\neg b}\\
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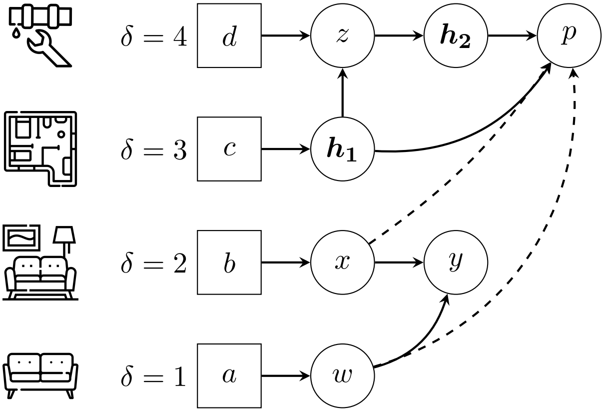

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\begin{aligned}
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$$

Trail Fixing will not Find the Implication [CFK24]

$$
\begin{aligned}
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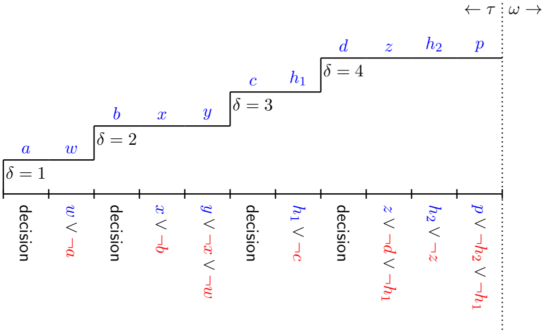

Var	λ
a	
b	
c	
d	
w	
x	
y	
z	
h_1	
h_2	
p	

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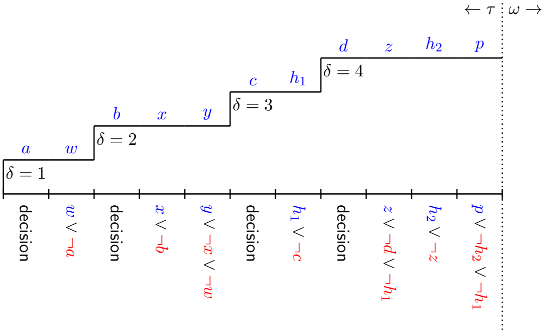

Var	λ
a	
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c	
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y	
z	
h_1	
h_2	
p	C_9

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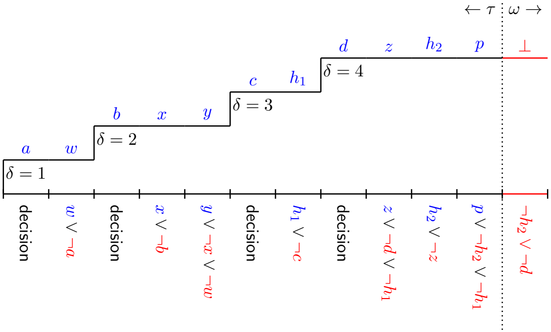
$$



Var	λ
a	
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h_2	
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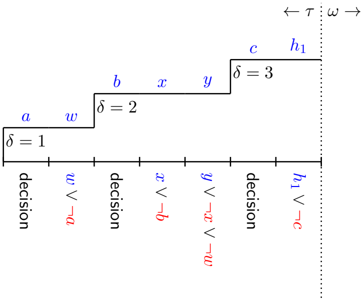

Var	λ
a	
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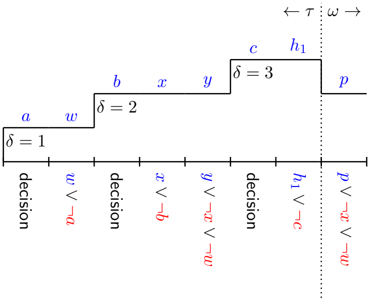


Var	λ
a	
b	
c	
d	
w	
x	
y	
z	
h_1	
h_2	
p	

Watched Literal Invariants

Let $\pi = \tau \cdot \omega$ be the current partial assignment, where τ is the propagated set.

Invariant (Watched Literals)

For each clause C in the formula watched by $c_1, c_2 \in C$, it must hold that:

$$\neg c_1 \in \tau \Rightarrow c_2 \in \pi$$

Watched Literal Invariants

Let $\pi = \tau \cdot \omega$ be the current partial assignment, where τ is the propagated set.

Invariant (Watched Literals)

For each clause C in the formula watched by $c_1, c_2 \in C$, it must hold that:

$$\neg c_1 \in \tau \Rightarrow c_2 \in \pi$$

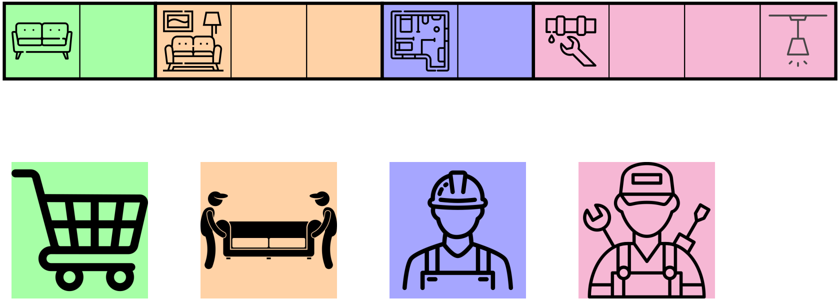
Let $\delta(\ell)$ be the decision level of literal ℓ in π
Let $\lambda(\ell)$ be the lazy reimplication clause of literal ℓ in π.

Invariant (Lazy backtrack compatible watched literals)

For each clause C in the formula watched by $c_1, c_2 \in C$, it must hold that:

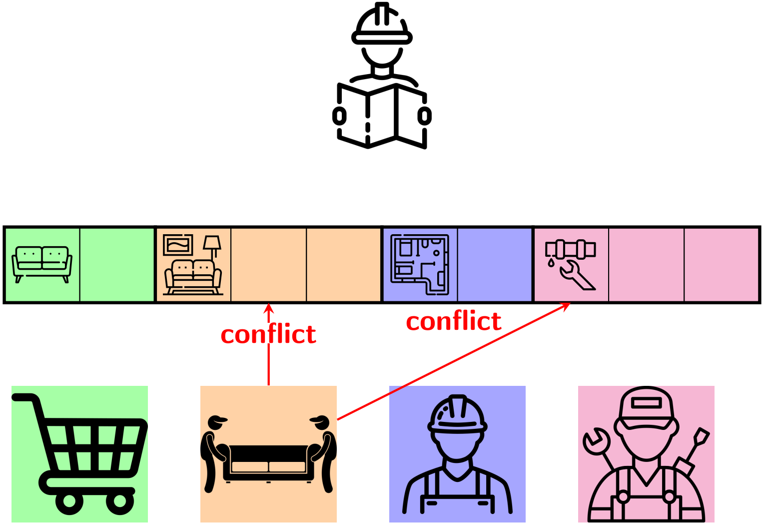
$$\neg c_1 \in \tau \Rightarrow \Big(c_2 \in \pi \wedge \big(\delta(c_2) \leq \delta(c_1) \vee \delta(\lambda(c_2) \setminus \{c_2\}) \leq \delta(c_1)\big)\Big)$$

We can Keep the House!

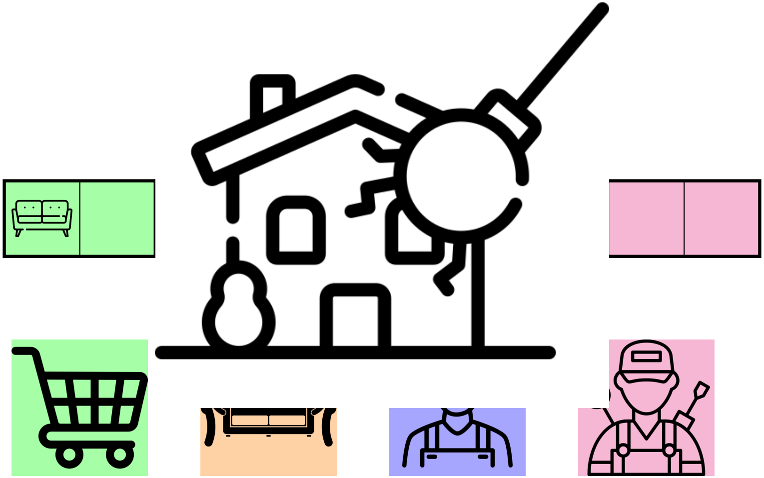


Part III: Graph Backtracking

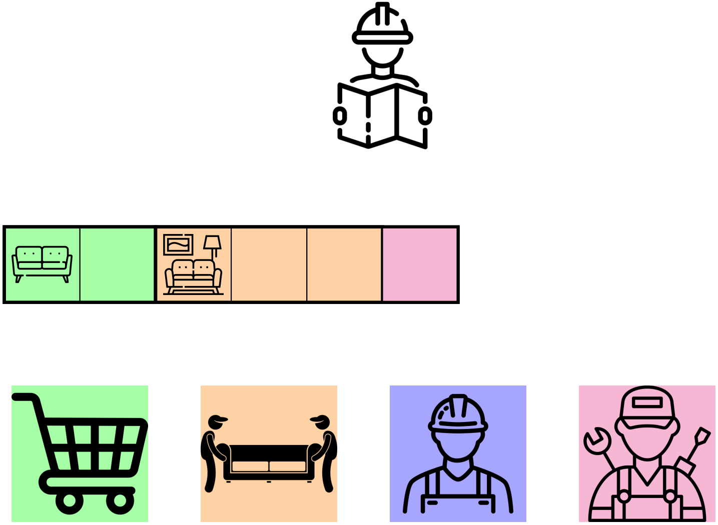
Graph Backtracking



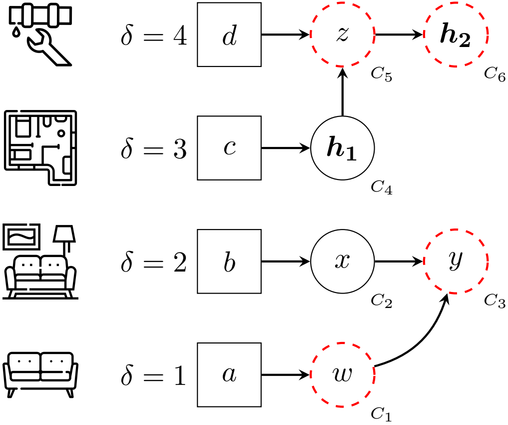
Graph Backtracking



Graph Backtracking



Non-Chronological Backtracking [SS99, MMZ$^+$01]



$C_1 = w \lor \neg a$

$C_2 = x \lor \neg b$

$C_3 = y \lor \neg w \lor \neg x$

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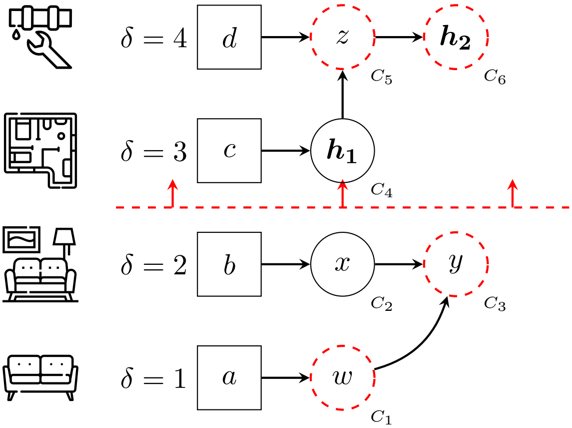
Non-Chronological Backtracking [SS99, MMZ+01]



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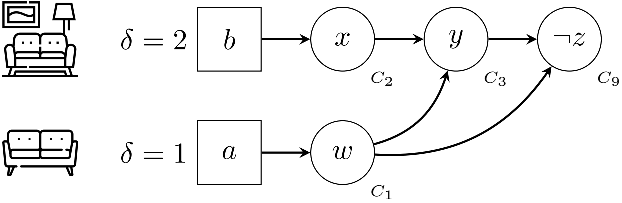
Non-Chronological Backtracking [SS99, MMZ+01]



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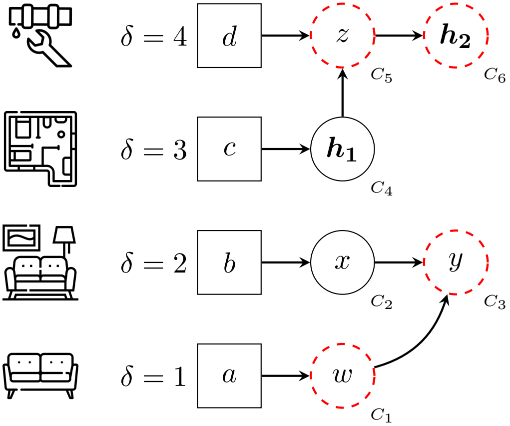
Chronological Backtracking [NR18, MB19, Nad22, CFK24]



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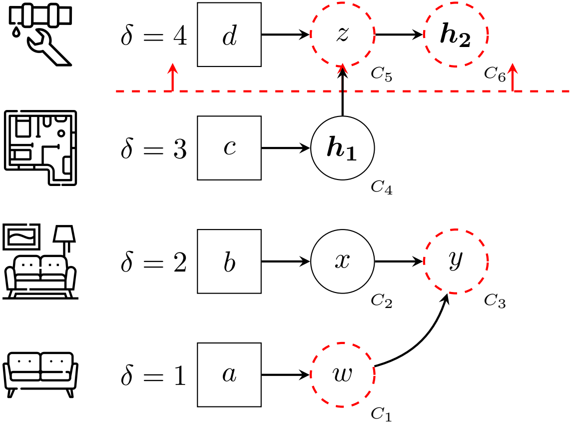

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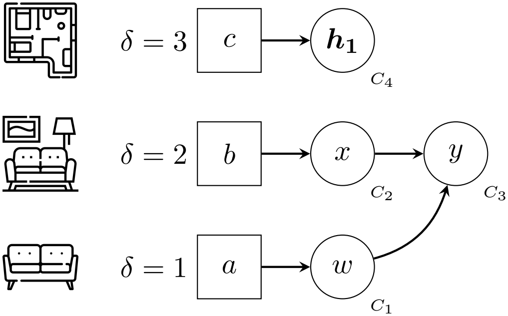

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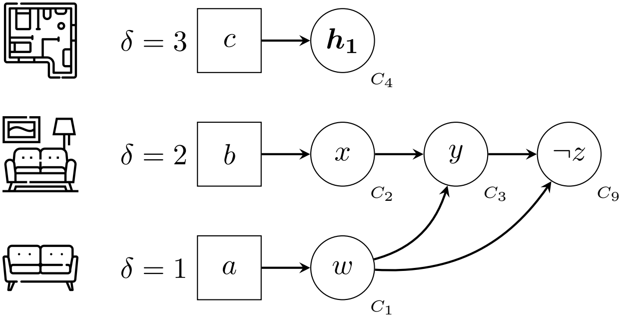

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\end{aligned}
$$

A More Surgical Architect

Extended Interface

We allow the user to provide a **cost** for each literal.

A More Surgical Architect

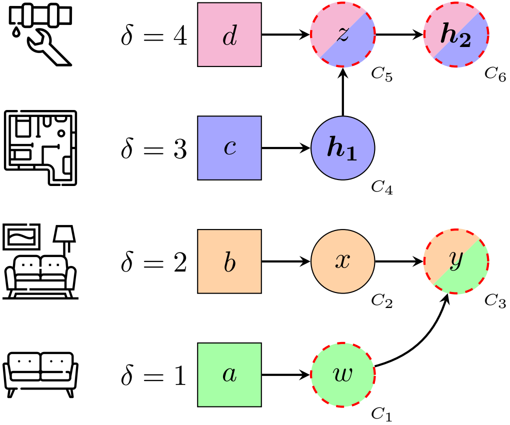
Extended Interface

We allow the user to provide a **cost** for each literal.

New Backtracking Scheme

Graph Backtracking (GB) searches the cheapest set of literals to backtrack to resolve a conflict. We use the implication graph to find such a set.

Graph Backtracking (GB)



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Graph Backtracking (GB)

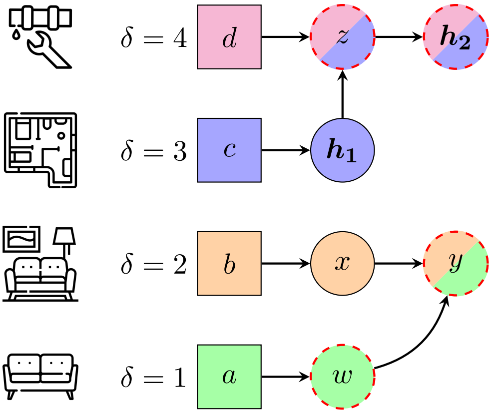
Backtrack options

option 1:

option 2:

option 3:

option 4:



Graph Backtracking (GB)

Backtrack options

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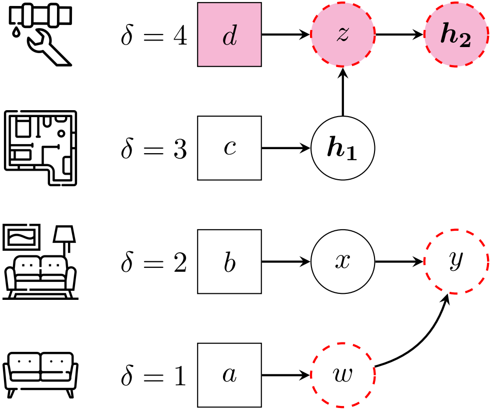

Graph Backtracking (GB)

Backtrack options

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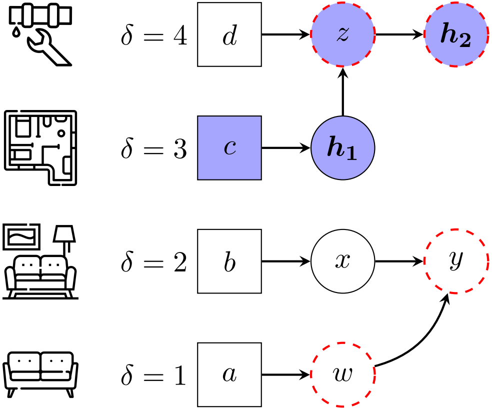

Graph Backtracking (GB)

Backtracking options

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option 2:
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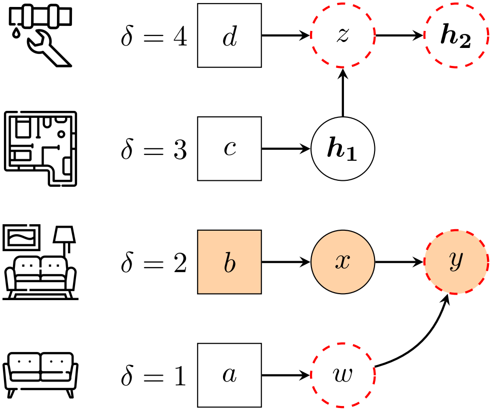

Graph Backtracking (GB)

Backtrack options

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option 2:

option 3:

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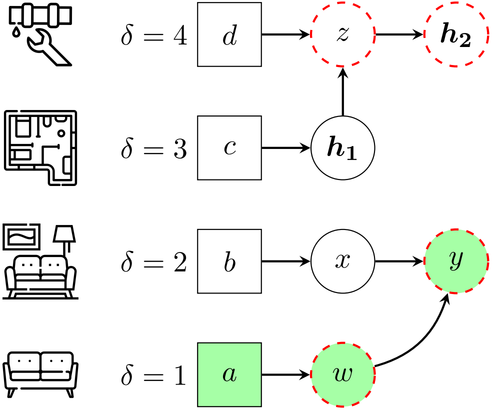
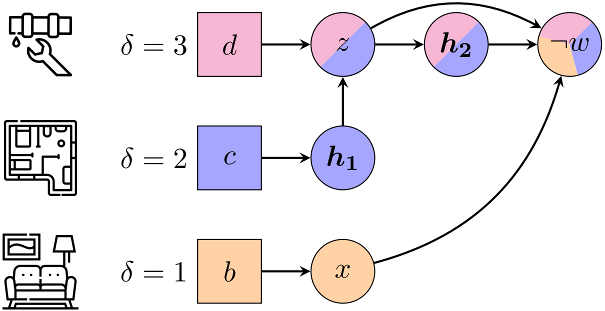


Graph Backtracking (GB)



What Graph Backtracking Buys Us

Precision

GB unassigns exactly the literals related to the conflict.

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User Control

The user-provided cost function allows for a more flexible backtracking strategy.

How is it done?

Simple enough...

Why is this not mainstream?

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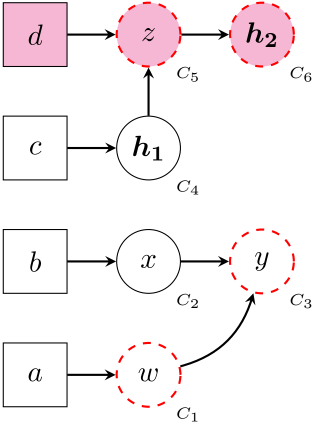
Why is this not mainstream?

Core differences with (N)CB

- Decision dependency tracking
- New Unique Implication Point (UIP) scheme
- Adjusted watch literals

First Unique Implication Point (UIP) with GB



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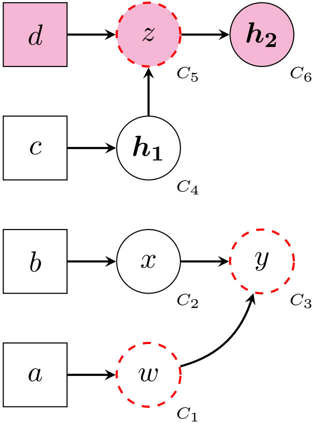
$C_8 = \neg w \lor \neg y \lor \neg z \lor \neg h_2$

First Unique Implication Point (UIP) with GB

$\neg w \vee \neg y \vee \neg z$



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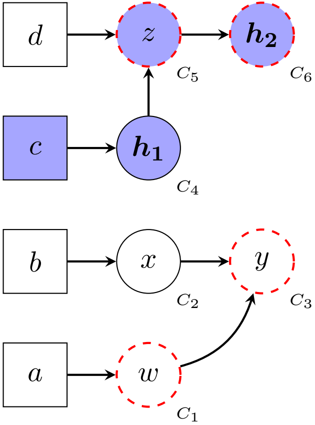

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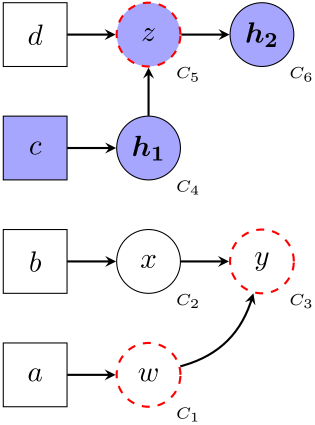

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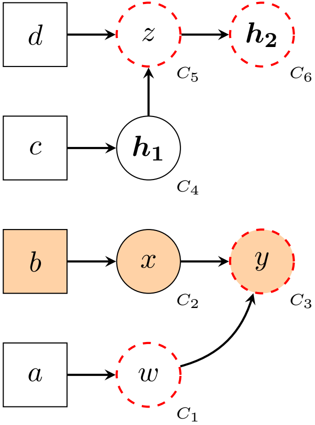

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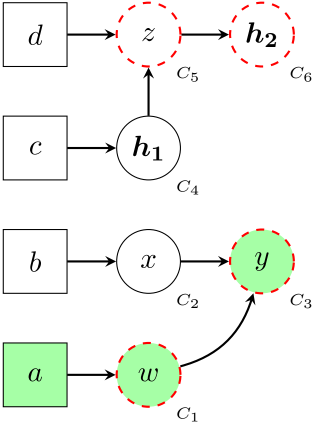

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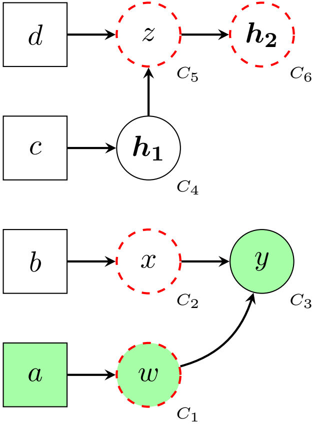

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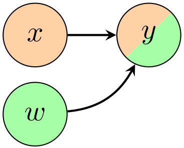
Watched Literals in Graph Backtracking



Let us watch y and $\neg x$ in C_3.

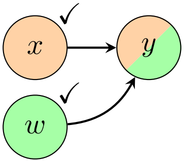
$$C_3 = \underline{y} \vee \underline{\neg x} \vee \neg w$$

Invariant (Watched Literals)

For each clause $C \in F$ watched by c_1, c_2, we have

$$\textit{propagated}(\neg c_1) \Rightarrow \textit{satisfied}(c_2)$$

Watched Literals in Graph Backtracking



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$$\text{propagated}(x) \Rightarrow \text{satisfied}(y)$$

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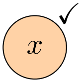

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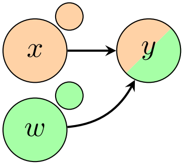
Watched Literals in Graph Backtracking (Solution)



We define cross-colors(ℓ) as the set of colors that trigger the repropagation of ℓ.

$$C_3 = \underline{y} \vee \underline{\neg x} \vee \neg w$$

Invariant (Watched Literals)

For each clause $C \in F$ watched by c_1, c_2, we have

$$\textit{propagated}(\neg c_1) \Rightarrow \left[\textit{satisfied}(c_2) \wedge \textit{colors}(c_2) \subseteq \textit{cross-colors}(c_1)\right]$$

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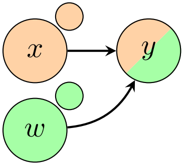

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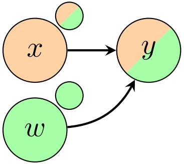

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Preliminary Empirical Results in NapSAT

Option	# Timeout	PAR2	Time (s)		Propagations $\times 10^6$	
NCB	16	380.7	106 ± 428	-	23.7 ± 46.4	-
CB	11	333.1	103 ± 414	97.33%	22.9 ± 42.8	96.30%
GB	11	299.7	72.4 ± 310	67.89%	12.6 ± 23.8	52.96%

Table: Empirical results on 1000 satisfiable random 3-coloring problems with 650 nodes and 1469 edges with 2-hour timeout.

Reminder

- NCB: Non-Chronological Backtracking
- CB: Chronological Backtracking with trail
- GB: Graph Backtracking

Results in Vampire [BBC+25] with AVATAR [Vor14]

We ran Vampire's AVATAR with NapSAT on 10617 TPTP problems requiring AVATAR.

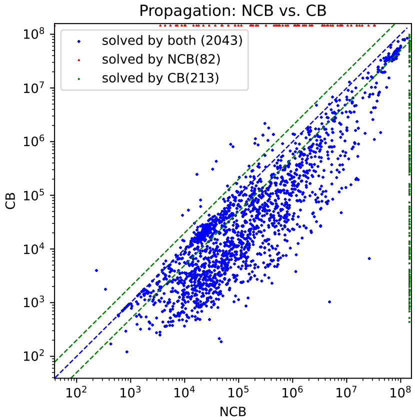

Figure: CB saves 66% of Boolean propagations compared to NCB

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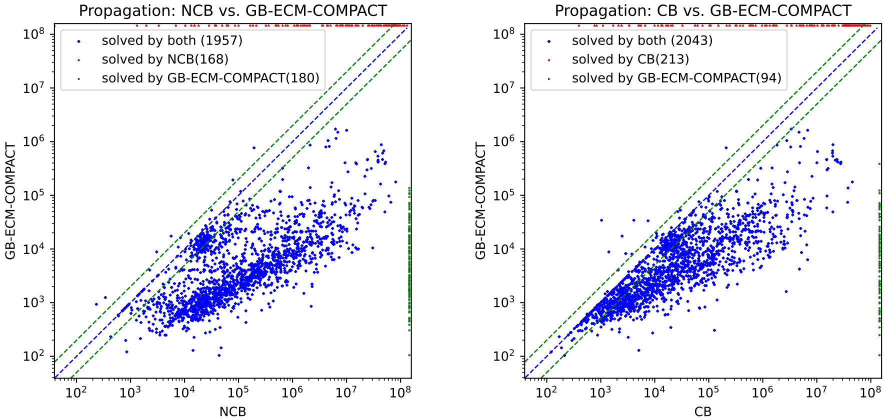

Figure: GB saves 98.5% of Boolean propagations compared to NCB, and 95.3% compared to CB

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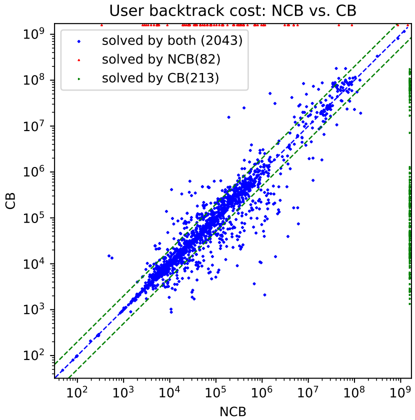

Figure: CB saves 2.2% of the estimated cost compared to NCB

Results in Vampire [BBC+25] with AVATAR [Vor14]

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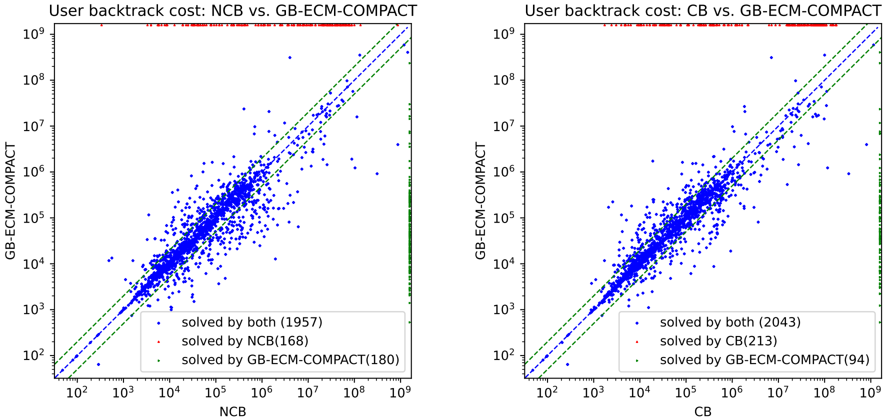

Figure: GB saves 42.1% of the estimated cost compared to NCB and 40.8% compared to CB

Results in Vampire [BBC+25] with AVATAR [Vor14]

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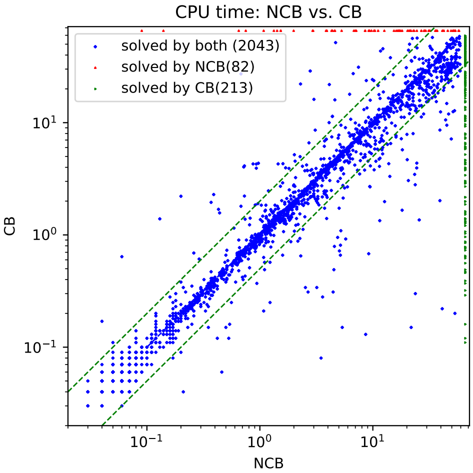

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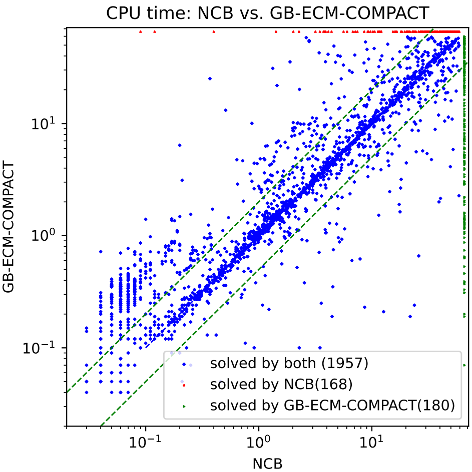

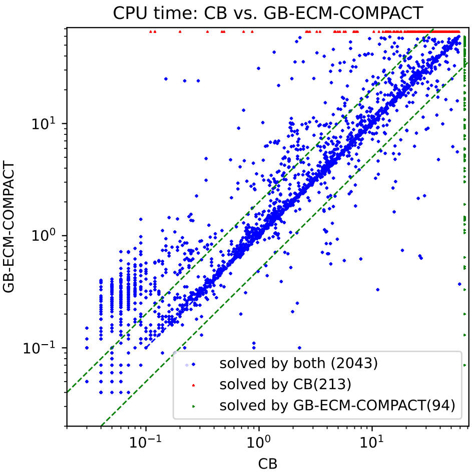

Conclusion

Summary

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- Incremental solving already benefits from smarter backtracking.
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- Relax SMT stack discipline.
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Thank you for your attention!

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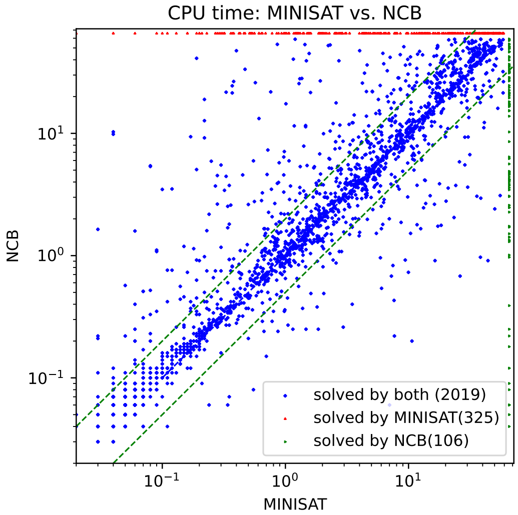
MiniSAT vs NapSAT in AVATAR



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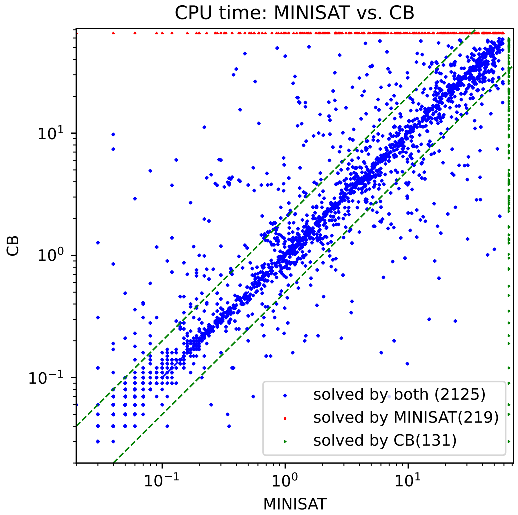

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